

Fundamental quantum physics in intense laser fields

Anton Ilderton

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ELI-NP summer school

anton.ilderton@chalmers.se



Stiftelsen Olle Engkvist Byggmästare
Signhild Engkvists Stiftelse



Links!

Link to my homepage, (extended) presentation and exercises

<https://goo.gl/e1F17f>

<https://sites.google.com/site/antonilderton/>

Outline

- QED
- Processes
- Strong field QED
- Solvable cases
- Examples

Our group and collaborators



Mattias
Marklund



Chris
Harvey



Arkady
Gonoskov



Greger
Torgrimsson



Felix
Mackenroth



Tom
Blackburn



Tom Heinzl
Plymouth, UK



Ben King
Plymouth, UK



Victor Dinu
Bucharest-Magurele

Stepan Bulanov
Ivan Gonoskov
Florian Hebenstreit
Alexander Sergeev
James Vary
Jonatan Wårdh
Xingbo Zhao

QED

- Field theory: electromagnetic field, electrons, positrons.

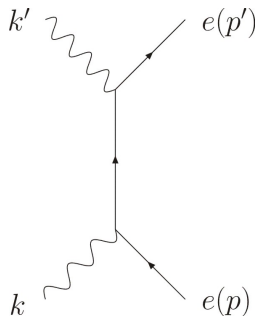
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi + \text{g.f.} - e\bar{\psi}A\psi + \text{counterterms}$$

ψ : electrons and positrons. A_μ : photons. Counterterms remove UV ∞ 's. Regularisation implicit.

- Tested to ridiculous precision.
- Describes all 'everyday' physics except gravity.
- Perturbation theory in $\alpha = e^2/(4\pi)$: asymptotic expansion.

Example: Compton scattering

- Feynman rules \rightarrow propagators, correlation functions.



- LSZ \rightarrow S -matrix, external leg wavefunctions $u_p e^{-ip \cdot x}$

Trees ...

● Nonlinear Compton scattering

Nikishov and Ritus, Sov.Phys.JETP 19 (1964) 529

Harvey, Heinzl, Ilderton PRA 79 (2009) 063407

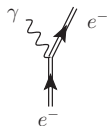
Boca and Florescu, PRA 80 (2009) 053403

Seipt, Kämpfer, PRA 83 (2011) 022101

Mackenroth, Di Piazza, PRA 83 (2011) 032106

Harvey, Heinzl, Ilderton, Marklund PRL 109 (2012) 100402

Dinu, Phys.Rev. A87 (2013) 052101



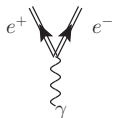
● Stimulated pair production

Nikishov and Ritus, Sov.Phys.JETP 19 (1964) 529

Heinzl, Ilderton, Marklund, Phys.Lett. B692 (2010) 250

Meuren, Keitel and Di Piazza, arXiv:1503.03271

Nousch, Seipt, Kämpfer, Titov. arXiv:1509.01983



● Cascades

Fedotov, et al. Phys.Rev.Lett. 105 (2010) 080402

Sokolov et al. Phys.Rev.Lett. 105 (2010) 195005

Elkina et al, Phys. Rev. ST Accel. Beams 14 (2011) 054401

Gonoskov, Ilderton et al., Phys. Rev. Lett. 111 (2013) 060404



...and loops

- Vacuum birefringence

Toll, PhD thesis, 1952

Heinzl et al., *Opt.Commun.* 267 (2006) 318.

Dinu, Heinzl, Ilderton, Marklund, Torgrimsson *PRD* 89 (2014) 125003

Karbstein, Gies, Reuter, Zepf *arXiv:1507.01084*



- Photon emission/splitting/scattering

Adler, *Annals Phys.* 67 (1971) 599

Lundström et al., *Phys.Rev.Lett.* 96 (2006) 083602

King and Keitel, *New J. Phys.* 14 (2012) 103002



- Schwinger pair production

Schwinger, *Phys. Rev.* 82 (1951) 664

Dunne, Gies, Schützhold, *PRL* 101 (2008) 130404

DiPiazza et al., *PRL* 103 (2009) 170403

Bulanov et al., *PRL* 104 (2010) 22040

Gonoskov, Ilderton et al., *PRL* 113 (2014) 014801

$$2 \operatorname{Im} \left[\text{Loop} \right] = \left| \begin{array}{c} e^+ \\ \text{Loop} \\ e^- \end{array} \right|^2$$

QED

- Field theory: electromagnetic field, electrons, positrons.

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi + \text{g.f.} - e\bar{\psi}A\psi + \text{counterterms}$$

ψ : electrons and positrons. A_μ : photons. Counterterms remove UV ∞ 's. Regularisation implicit.

- Our interest: a **strong background field** is present.

When is a field “strong”?

- What do we mean by a **strong field**?

When is a field “strong”?

- What do we mean by a **strong field**?
- Field of “typical” strength E , frequency ω :

$$\text{Lorentz} \quad \longrightarrow \quad x'' \sim \frac{eE}{m\omega} x'$$

- Coupling to the external field is

$$\eta = \frac{eE}{m\omega}$$

- Relativistic effects: $\eta > 1 \implies eE\lambda > m$
- Nonlinear/‘multiphoton’ effects: $\eta > 1 \implies eE\lambda_C > \omega$
- Strong fields $\implies \eta > 1 \implies$ **no perturbation in η .**

Coherent states (how to include a background field)

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi + \text{g.f.} - e\bar{\psi}A\psi + \text{counterterms}$$

ψ : electrons and positrons. A_μ : photons. Counterterms remove UV ∞ 's. Regularisation implicit.

Scattering in background fields ...

... is scattering between coherent states.

e.g. Compton: $\langle e(p'), \gamma(k'), C | \hat{S}_{\text{QED}} | e(p), \gamma(k), C \rangle$

- Same state \implies **beam depletion** neglected!
- Replacement rule: $eA_\mu \rightarrow eA_\mu + eA_\mu^{\text{ext}}$.

Strong Field QED

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \text{g.f.} + \bar{\psi}(i\mathcal{D} - m)\psi \\ - e\bar{\psi}A\psi + \text{counterterms}$$

-
- $\mathcal{D}_\mu = \partial_\mu + ieA_\mu^{\text{ext}}$: background covariant derivative.

Strong Field QED

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \text{g.f.} + \bar{\psi}(i\mathcal{D} - m)\psi - e\bar{\psi}A\psi + \text{counterterms}$$

- $\mathcal{D}_\mu = \partial_\mu + ieA_\mu^{\text{ext}}$: background covariant derivative.

- Usual photon propagator.



- Fermion propagator: $(i\mathcal{D} - m)^{-1}$



Exact treatment of η effects. Describes Lorentz force motion

- Vertex: two fermions, one photon as usual.

Quantum emission and recoil contained in vertex

- “Furry picture expansion”.

Do we know $(i\mathcal{D} - m)^{-1}$? Laser field models

- Monochromatic plane waves.

Nikishov, Ritus, Narozhny 1964

- Pulsed plane waves: **finite size effects**.

Boca and Florescu, Phys.Rev. A 80 (2009) 053403

Heinzl, Ilderton, Marklund, Phys.Lett. B 692 (2010) 250

Heinzl, Seipt and Kämpfer, Phys.Rev. A 81 (2010) 022125

Mackenroth & Di Piazza, Phys.Rev. A 83 (2011) 032106

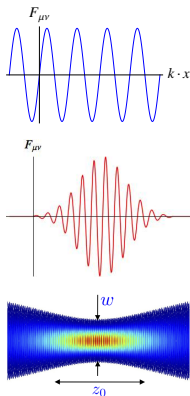
- Realistic beams

No corresponding exact treatment.

- ▷ Use plane waves...

$$F_{\mu\nu} \equiv F_{\mu\nu}(k.x), \quad k^2 = 0 \quad k.x \equiv k_+ x^+ \equiv \omega(x^0 + x^3).$$

- 'One dimensional'. **Lightfront**. Neville & Rohrlich, Phys. Rev. D 3 1692 (1971)



Calculations in plane waves: classical

Classical motion: identify momentum π_μ and position x^μ .

Calculations in plane waves: classical

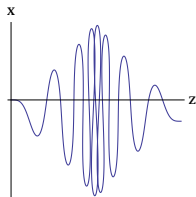
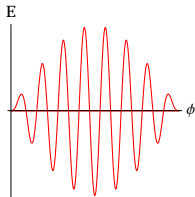
Classical motion: identify momentum π_μ and position x^μ .

- Transversality of field \implies three conserved quantities.

\implies Use phase $\phi := k \cdot x = \omega(x^0 + x^3)$ to parameterise.

- Initial $p_\mu \rightarrow \pi_\mu(\phi)$ depending on

- $$a_{1,2} = \int_{-\infty}^{\phi} d\varphi \frac{e}{\omega} E_{1,2}(\varphi)$$



Calculations in plane waves: quantum

- LSZ \implies in/out wavefunctions. (Using potential $eA_\mu^{\text{ext}} = a_\mu \cdot$)

$$e^- \text{ in: } \Psi_p^-(x) = \left[\mathbb{1} + \frac{1}{2k \cdot p} \not{k} \not{a}(k \cdot x) \right] u_p e^{-ip \cdot x - \frac{i}{2k \cdot p} \int_{-\infty}^{k \cdot x} 2a \cdot p - a^2}$$

- (Solutions of Dirac equation $i\not{D} - m = 0$.)
-

What does all this mean?

Calculations in plane waves: quantum

- LSZ \implies in/out wavefunctions. (Using potential $eA_\mu^{\text{ext}} = a_\mu \cdot$)

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- (Solutions of Dirac equation $i\not{D} - m = 0$.)
-

What does all this mean?

no background : $u_p \exp(-ip \cdot x) = u_p \exp(-i\not{D}^{-1} \cdot p)$

with background : $u_{\pi(\phi)} \exp[-i\not{D}^{-1} \cdot \pi(\phi)]$

- Exact solutions due to **conserved quantities**
- Simplicity due to **vanishing of field invariants**: no pairs!

Example: stimulated pair production

- Incoming photon creates a **pair**.

$$\underset{\text{laser}}{\gamma(k')} \longrightarrow e^-(p) + e^+(p')$$

$$k'_\mu + (\text{background}) = p_\mu + p'_\mu$$

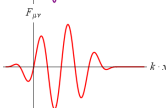
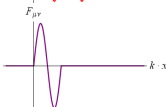
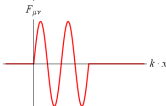
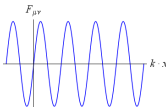
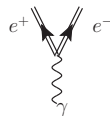
- S -matrix element:

$$S_{fi} = -ie \int d^4x \Psi_p^{\text{out}-}(x) e^{-ik' \cdot x} \not{\epsilon} \Psi_{p'}^{\text{out}+}(x)$$

- For normalisations, flux factors etc see appendix in:

Ilderton and Torgrimsson, *Phys.Rev.D* 87 (2013) 085040 [arXiv:1210.6840]

- Periodic** vs. **short pulse** backgrounds.



Old results: very long pulses.

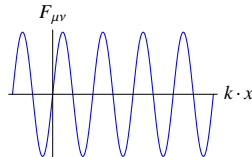
- Periodic waves. **Infinite** duration.

Nikshov, Ritus, *Sov.Phys.JETP* 19 (1964) 529

- Charges: rapid quiver motion.

→ Quasi-momentum q_μ

$$q_\mu = p_\mu + \frac{\eta^2}{2k \cdot p} k_\mu$$



Old results: very long pulses.

- Periodic waves. **Infinite** duration.

Nikshov, Ritus, Sov.Phys.JETP 19 (1964) 529

- Charges: rapid quiver motion.

→ Quasi-momentum q_μ

$$q_\mu = p_\mu + \frac{\eta^2}{2k \cdot p} k_\mu$$

- S-matrix supported on:

$$k'_\mu + n k_\mu = q_\mu + q'_\mu$$

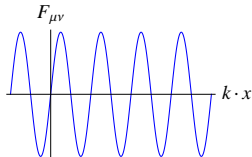
$$q^2 = m_*^2 \equiv m^2(1 + \eta^2).$$

Sengupta, 1952

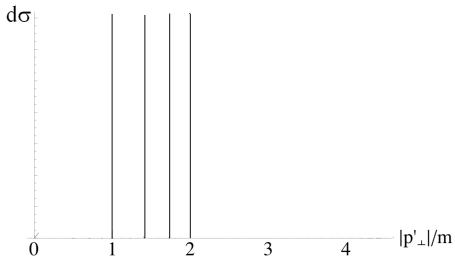
? Pair production threshold: $n > \frac{2m_*^2}{k \cdot k'}$

- ELI optical laser: $m_* \sim 10^2 m!$

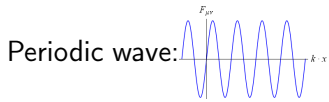
Extreme Light Infrastructure



The role of the shifted mass



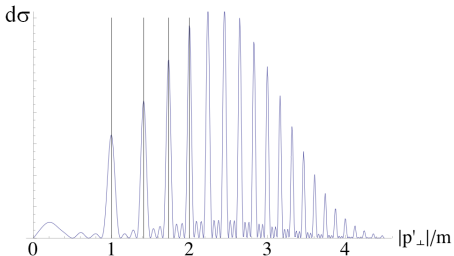
- Differential cross-section.
- (arb. units)
- Energies: $0 \sim m$ and $1 \sim m_*$.



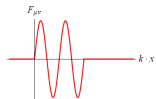
Periodic wave:

- Line spectrum.
- Shifted threshold.
- $k.k' > 2m^2(1 + \eta^2)$

The role of the shifted mass

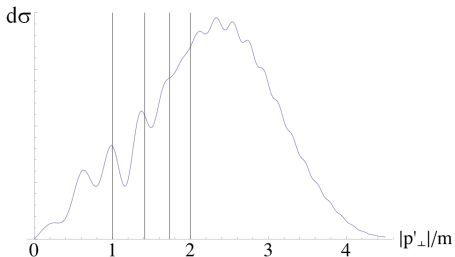


Wavetrain:
few cycles

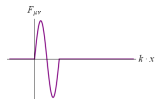


- Substructure.
- Sub-threshold behaviour
- Peaks = **resonances**.
- Lightfront momentum transfer:
- Peaks when average = **multiple** of driving frequency ω .
- **Resonances!** Just looks like energy to produce m_* pairs.
- Identifies $q_\mu = \langle \pi_\mu \rangle$

The role of the shifted mass



Wavetrain:
one cycle

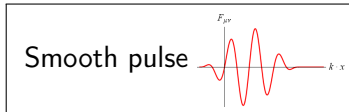
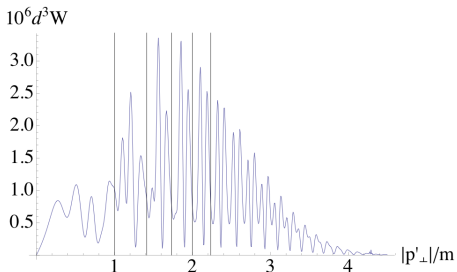


- Different structure.
- Clear signal between m and m_* .

⇒ Mass ↔ shifted mass dominance.

- m_* not 'in control'.
- The emission spectrum reflects the pulse shape.

The role of the shifted mass



- Different structure.
- Shifted peaks.
- **Messy!**

- $m_* \rightarrow M(k.x, k.y)$.

Kibble, Salam, Strathdee, NPB 96 (1975) 255

- Different pulse shape: there exist other effective masses m_* !

Harvey, Heinzl, Ilderton, Marklund Phys.Rev.Lett. 109 (2012) 100402

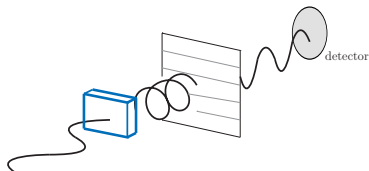
Optics: birefringence

- A birefringent medium (length L).
- Refractive indices n_{\parallel} and n_{\perp} .
- A beam of light (wavelength λ').

⇒ Effective probe velocity depends on polarisation.

- Linear \rightarrow elliptic, ellipticity δ :

$$\delta = \pi(n_{\parallel} - n_{\perp})\frac{L}{\lambda'}$$



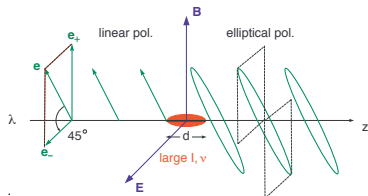
Vacuum birefringence

The quantum vacuum, exposed to intense light, is also birefringent.

Toll, PhD thesis, 1952

- Intense optical laser
- Send in X-ray probe: **linear pol**
- Probe emerges: **elliptical pol**
- Needs accurate X-ray polarimetry

Marx et al PRL 110 (2013)



T. Heinzl et al, Opt. Commun. 267 (2006) 318

- **Virtual loops** → refractive indices.
 - First measurement of light-by-light?
- Halpern, Phys.Rev. 44 (1934), HIBEF, ELI-NP
- Beyond the SM? What runs the loop?



Scattering → observables

- Beam ellipticity: photons flipping **helicity-state**.

- Ellipticity: $\delta^2 = \mathbb{P}(\text{flip}) = |S_{flip}|^2$

- **Much simpler** than going 'via indices'.

Dinu, Ilderton, Heinzl, Marklund, Torgrimsson, PRD 90 (2014) & PRD 89 (2014)

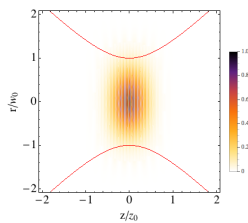
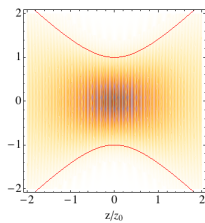
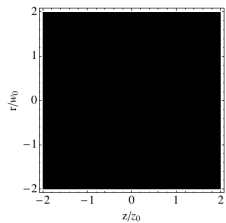
- Look at impact of realistic pulse geometry.

! Precision expt.

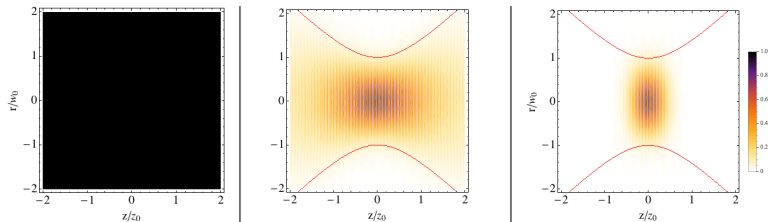
! Need to do better than plane waves.

- Build up complexity.

Impact of pulse geometry

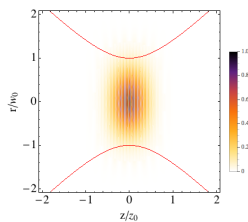
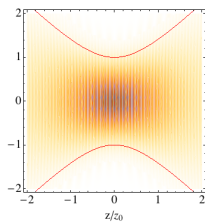
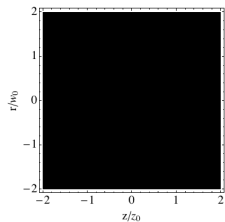


Impact of pulse geometry



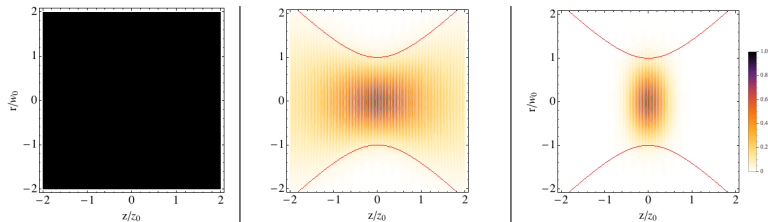
$$\delta = \frac{2\alpha}{15} \frac{E^2}{E_S^2} \frac{2z_0}{\lambda'}$$

Impact of pulse geometry



$$\delta = \frac{2\alpha}{15} \frac{E^2}{E_S^2} \frac{2z_0}{\lambda'} \times \frac{\pi}{4}$$

Impact of pulse geometry

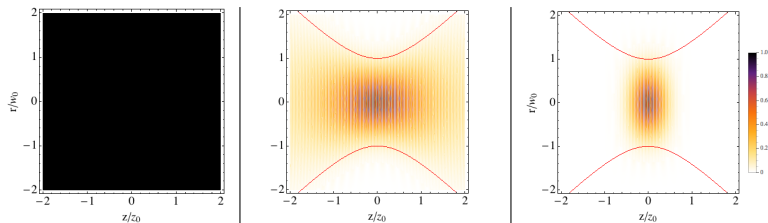


$$\delta = \frac{2\alpha}{15} \frac{E^2}{E_S^2} \frac{2z_0}{\lambda'} \times \frac{\pi}{4} \times \frac{1}{4\sqrt{\pi \log 2}} \frac{\tau}{z_0}$$

- Hibef @ European X-FEL:

En. = 30J, $\lambda = 800\text{nm}$, $w_0 = 2\mu\text{m}$, $\tau = 30\text{fs}$ & $\lambda' = 0.1\text{nm}$

Impact of pulse geometry



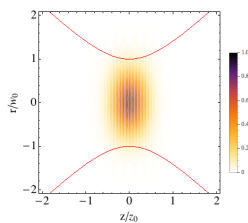
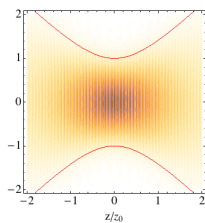
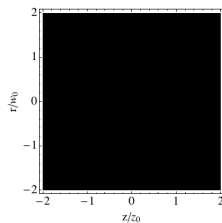
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$$\delta = 2.2 \times 10^{-5}$$

Impact of pulse geometry



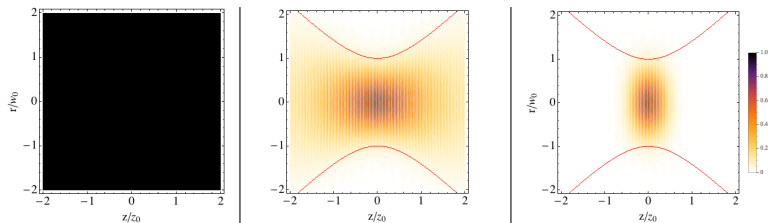
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- Hibef @ European X-FEL:

En. = 30J, $\lambda = 800\text{nm}$, $w_0 = 2\mu\text{m}$, $\tau = 30\text{fs}$ & $\lambda' = 0.1\text{nm}$

$$\delta = 2.2 \times 10^{-5} \quad \rightarrow \quad 1.7 \times 10^{-5}$$

Impact of pulse geometry



$$\delta = \frac{2\alpha}{15} \frac{E^2}{E_S^2} \frac{2z_0}{\lambda'} \times \frac{\pi}{4} \times \frac{1}{4\sqrt{\pi \log 2}} \frac{\tau}{z_0}$$

- Hibef @ European X-FEL:

En. = 30J, $\lambda = 800\text{nm}$, $w_0 = 2\mu\text{m}$, $\tau = 30\text{fs}$ & $\lambda' = 0.1\text{nm}$

$$\delta = 2.2 \times 10^{-5} \quad \rightarrow 1.7 \times 10^{-5} \quad \rightarrow 2.0 \times 10^{-6}$$

- Only slightly beyond what can be measured today...

Birefringence at ELI-NP

- Pump/target: an intense (PW) optical laser
- Probe: **synchrotron emission** from laser-electron collisions.

$$\delta = \frac{2\alpha}{15} \frac{E^2}{E_S^2} \frac{2z_0}{\lambda'}$$

- ☺ Strong laser fields (E/E_S is 'large')
- ☺ Synchrotron radiation: gamma rays (λ' small)
- ☹ Messy environment, lose precision.
- ☹ Gamma ray spectroscopy is challenging!

Conclusions

- If quantum and relativistic effects are important:

QFT is the theory to use.

- Interested in theory?

All-orders and nonperturbative physics

Back-reaction, realistic fields...

- Interested in experiment?

Unobserved standard model processes

Beyond the standard model

Links!

Link to my homepage, (extended) presentation and exercises

<https://goo.gl/e1F17f>

<https://sites.google.com/site/antonilderton/>